Step-Twist-Junction Waveguide Filters*

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Summary—**The properties of step-twist-junction discontinuities in rectangular waveguide are considered. Methods are presented whereby these step-twist-junctions may be used in filter design and, in particular, in the design of variable bandwidth constant-resonant frequency filters.**

INTRODUCTION

IN MANY laboratories work is often concentrated in some particular frequency band and, in such cases, a series of filters of different bandwidths tuned in this band are often a necessity as well as a great convenience.

This paper presents the results of design work on filters utilizing step-twist-junction discontinuities[1] and on filters utilizing modified step-twist-junction discontinuities. The former presents a particularly simple structure which can be easily fabricated, while the latter presents the useful property of variable bandwidth with fixed resonant frequency. This property allows one to adjust one filter for a wide range of bandwidths and thus avoid the tedious design of many separate filters.

DESIGN THEORY

A step-twist-junction will be defined to be a rectangular-to-rectangular waveguide junction of the type that would be produced by electroforming on the mandrel as illustrated in Fig. 1 and then removing the mandrel.

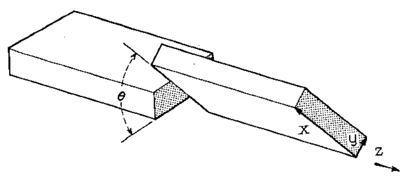

Fig. 1—Mandrel for a step-twist-junction.

The longitudinal axes of the two blocks will be assumed colinear and their cross-sectional dimensions identical. Thus, only one parameter θ will be needed to specify the junction completely. θ will be defined as the angular twist of the junction as illustrated in Fig. 1. In this paper the right-or-left-handedness of θ is irrelevant and, hence, we will specify $0 \leq \theta \leq \pi/2$, *i.e.*, θ will always be positive and will be the acute angle between the x, z faces of the two rectangular blocks.

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[1] H. A. Wheeler and H. Schwiebert, "Step-twist waveguide components," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 44–52; October, 1955.

If a guided wave of frequency 11.178 kMc is introduced on one side of a step-twist-junction in 0.400 inch $\times$0.900 inch guide and the guide on the other side of the junction is terminated with the characteristic impedance of the guide, an examination of the admittance of the step-twist-junction plus termination by means of a standing-wave detector produces the plot presented in Fig. 2 as θ is varied.

The information contained in Fig. 2 is enough to design a filter using two step-twist-junctions provided that use is made of further information obtainable from Wheeler and Schwiebert.[1]

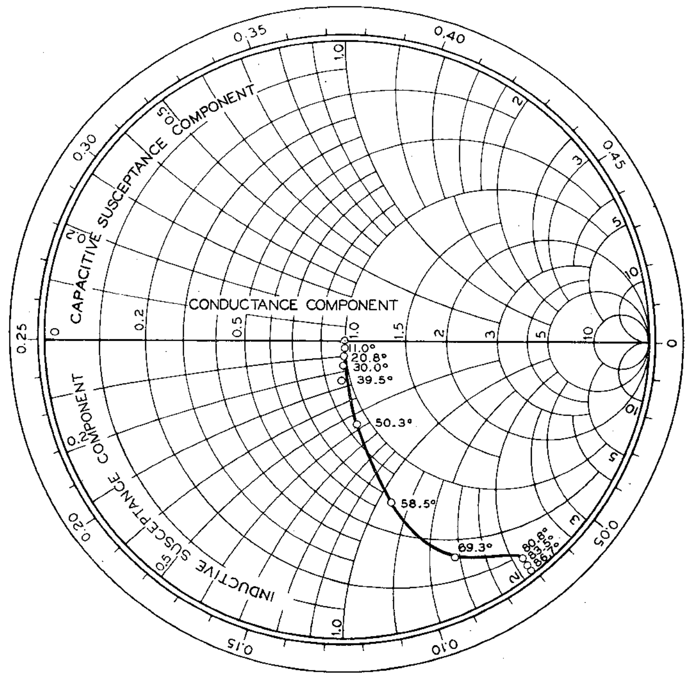


Fig. 2—The range of admittance of a step-twist-junction at 11.178 kMc for $0° \leq \theta \leq 90°$.

The following observation can be made. Any two-port passive symmetrical lossless microwave network may be represented at one frequency by a pure shunt susceptance with two equal lengths of transmission line on either side, as illustrated in Fig. 3(a). If this simple type of equivalent circuit is assumed for a step-twist-junction, it might be expected, in general, that the functional relation of length, susceptance, and frequency would assume some complicated form. Actually one finds, experimentally (see Appendix), that for a considerable range of frequencies in the vicinity of 11.178 kMc, the lengths of line may be accurately expressed as $L = a\lambda_g$ with a some constant, and the susceptance B as a con-

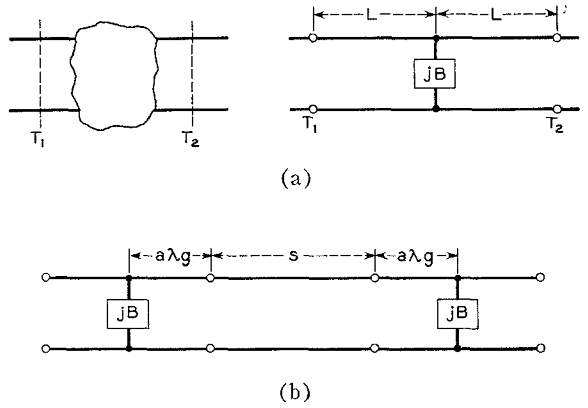


Fig. 3—(a) A two-port (symmetrical) microwave network and a transmission-line equivalent circuit. Planes T_1 and T_2 are assumed to be out of the area of fringing fields. (b) Transmission-line equivalent circuit for a step-twist-junction waveguide filter.

stant. This approximation will be made in all the succeeding work.

The lengths of line and the value of the susceptance for the equivalent circuit may be found graphically or analytically. The graphical method consists of drawing a constant standing-wave ratio circle (towards the load) from the point on the admittance chart (Fig. 2) representing the twist for the particular value of θ and λ_g, to an intersection point with the unit conductance circle. The magnitude of the equivalent pure shunt susceptance is read directly from this intersection, while the lengths of line are such as to have a equal the fractional wavelength separation of the unit circle intersection and the starting point above. (The shunt susceptance will turn out inductive if the minimum length of line is used.) These values can be equivalently calculated directly from the measurements of standing-wave ratio and minima positions from which Fig. 2 was obtained, and they are presented in Figs. 4 and 5.

Having obtained the equivalent circuit parameters, we will now proceed to develop expressions for the design of step-twist-junction waveguide filters in terms of resonant frequency wavelength and loaded Q.

For the usual type of filter, *i.e.*, two frequency independent pure shunt susceptances jB' separated by a distance l, the design formulas are obtained from Mumford[2] as

$$\tan \frac{2\pi l}{\lambda_{g0}} = \frac{2}{B'}, \tag{1}$$

$$Q = \left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \frac{\dfrac{2\pi l}{\lambda_{g0}}}{2 \sin^{-1} \dfrac{2}{\sqrt{(B')^4 + 4(B')^2}}}, \tag{2a}$$

[2] W. W. Mumford, "Maximally flat filters in waveguide," *Bell Sys. Tech. J.*, vol. 27, pp. 684–713; October, 1948.

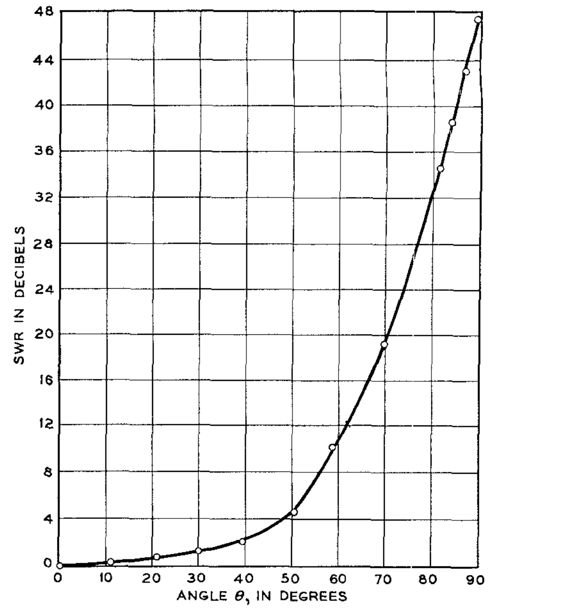


Fig. 4—SWR vs θ for a step-twist-junction with f=11.178 kMc.

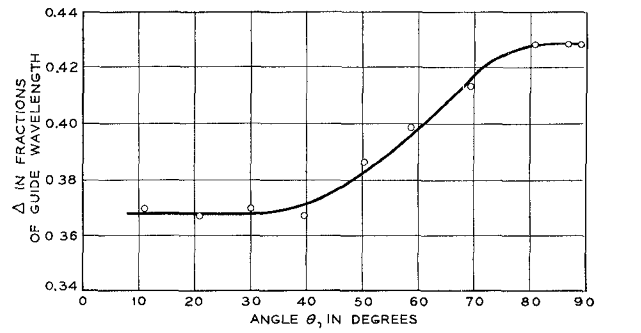


Fig. 5—Fractional wavelength separation of the step-twist-junction and the nearest minimum (toward the generator) of the standing-wave pattern of Fig. 2.

or

$$Q = \left(\frac{\lambda_{g0}}{\lambda_0}\right) \frac{\tan^{-1} \dfrac{2}{B'}}{2 \sin^{-1} \dfrac{2}{\sqrt{(B')^4 + 4(B')^2}}}, \tag{2b}$$

where

Q ≡ the loaded $Q \equiv f_0/f_1 - f_2$ with f_0, f_1, and f_2 being the resonant frequency and the upper and lower half-power frequencies, respectively,[3]

λ_{g0} ≡ the resonant frequency wavelength in the guide,

[3] Mumford's formulas are actually written in terms of a so-called wavelength Q defined by

$$Q = \frac{\dfrac{1}{\lambda_0}}{\dfrac{1}{\lambda_{g2}} - \dfrac{1}{\lambda_{g1}}},$$

where λ_0, λ_{g1}, and λ_{g2} are the resonant frequency-guide wavelength and the upper and lower half-power guide wavelengths, respectively. In (2a) and (2b) Mumford's equations have been rewritten in the more conventional loaded Q defined in terms of frequencies.

$\lambda_0 \equiv$ the resonant frequency wavelength in free space. B' is positive or negative when the susceptance is capacitive or inductive, respectively,

These formulas will now be adapted to a twist filter. The necessity for this modification arises because the effective separation of our susceptances (the pure ones of our equivalent circuit) may be written [see Fig. 3(b)]

$$d = 2a\lambda_g + s, \tag{3}$$

where s is the physical separation of the two twist-junctions and $2a\lambda_g$ is a length of line added by the equivalent circuits of the twist-junctions. Thus for a twist filter (1) becomes

$$\tan \frac{2\pi}{\lambda_{g0}} (2a\lambda_{g0} + s) = \frac{2}{B}, \tag{4}$$

where it must be remembered that B is negative.

The expression for the loaded Q of a filter may be obtained[2] from the equivalence of two shunt susceptances jB', separated by a distance l, to a parallel resonant circuit (and a length of line to adjust phases). The admittance of the tuned circuit jB_x can be written as[2]

$$jB_x = jB'(B' \sin \theta_1 - 2 \cos 2\theta_1), \tag{5}$$

where

$$\theta_1 = \frac{2\pi l}{\lambda_g}.$$

The half-power points may be shown to occur at $B_x = \pm 2$. For the twist filter, the use of (3) and (4) in (5), in combination with double angle formulas, yields for the half-power wavelengths λ_{g1} and λ_{g2}

$$\frac{1}{\lambda_{g1}} = \frac{1}{\lambda_{g0}} - \frac{1}{2\pi s} \sin^{-1} \frac{2}{\sqrt{B^4 + 4B^2}}$$

$$\frac{1}{\lambda_{g2}} = \frac{1}{\lambda_{g0}} + \frac{1}{2\pi s} \sin^{-1} \frac{2}{\sqrt{B^4 + 4B^2}}, \tag{6}$$

and writing

$$Q = \left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \frac{\frac{1}{\lambda_{g0}}}{\frac{1}{\lambda_{g2}} - \frac{1}{\lambda_{g1}}},$$

we have

$$Q = \left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \frac{\frac{2\pi s}{\lambda_{g0}}}{2 \sin^{-1} \frac{2}{\sqrt{B^4 + 4B^2}}}. \tag{7}$$

Note the similarity of (7) with (2a), but also observe that the next step [Eq. (8)]

$$Q = \left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \left[\frac{\tan^{-1} \frac{2}{B}}{2 \sin^{-1} \frac{2}{\sqrt{B^4 + 4B^2}}} - \frac{4\pi a}{2 \sin^{-1} \frac{2}{\sqrt{B^4 + 4B^2}}} \right] \tag{8}$$

is different from (2b). From (7) we observe that s, the physical separation of the step-twist-junctions, occurs in the expression for the loaded Q as does l in (2a) for pure susceptive irises. That this should be the case can be seen from the fact that the phase separation of the equivalent circuit susceptances, *i.e.*, $2\pi d/\lambda_g$ is $2\pi[2a + (s/\lambda_g)]$, and only the s/λ_g term is frequency sensitive.

Thus (4) and (8) are the design equations for a step-twist-junction waveguide filter corresponding to (1) and (2b) for the usual type of filter. The design procedure for a step-twist-junction waveguide filter is as follows. First a curve such as that in Fig. 2 must be obtained (by extrapolation or measurement). Then for a given Q and resonant frequency, the value of B (and the corresponding value of a determined from the Smith chart plot) must be found that will satisfy (8). These values of B and a must then be substituted into (4) to determine s, the physical separation of the two step-twist-junctions. The value of θ is obtained from the curve corresponding to Fig. 4 by finding that value of θ which corresponds to a standing-wave ratio, identical to the one obtained from the pure susceptance B backed by a termination. From (4) we observe that for a particular resonant frequency the twist filter will be shorter for a given loaded Q than one made with pure inductive irises. The magnitude of this shortening depends upon the frequency of operation and the desired loaded Q.

Step-twist-junction waveguide filters can be designed with a rotatable center section. If different types of thin irises are attached to a step-twist-junction, the admittance vs θ plot for this junction and thus the resonant frequency-bandwidth plot for a filter made from two of the junctions can be radically altered. Note in particular that if the admittance plot of Fig. 2 lay along a straight line passing through the center of the chart, then for a filter, the design separation s for a given resonant frequency would be the same for any bandwidth. An admittance plot at 11.178 kMc for a step-twist-junction with a 0.010-inch thick 0.400-inch diameter centered circular iris inserted in the plane of the junction is presented in Fig. 6. This particular combination is observed to trace out the desired straight line as θ is varied from

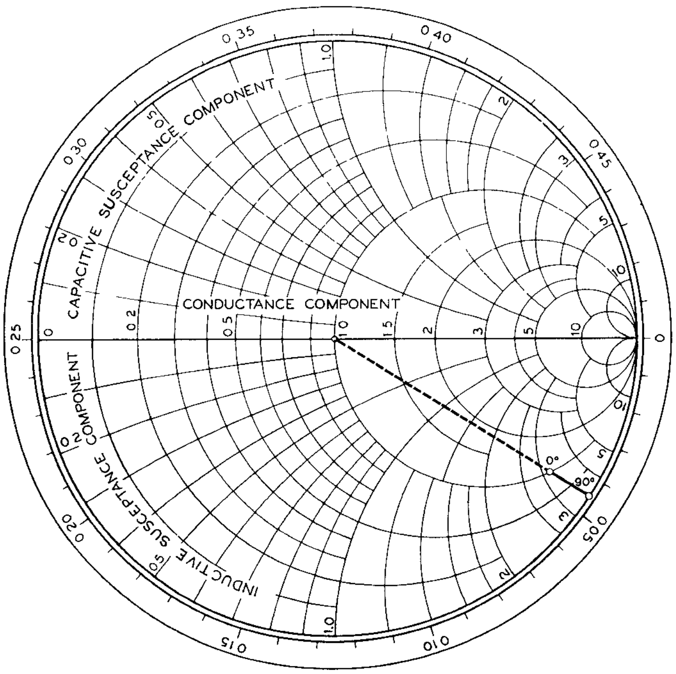


Fig. 6—The range of admittance of a step-twist-junction with centered 0.400-inch diameter iris at 11.178 kMc for $0° \leq \theta \leq 90°$.

90° (total reflection) to 0° (intersection with unit circle). Thus a filter composed of two such junctions should have the property of variable bandwidth and constant resonant frequency as θ is varied.

Experimental Results

A frequency vs transmission loss curve is presented in Fig. 7 for a singe-cavity step-twist filter. This cavity was designed for $f_0 = 11.72$ kMc and a 3-db bandwidth of 154 Mc. Experimentally we obtained $f_0 = 11.58$ kMc and a 3-db bandwidth of 147 Mc. The closeness of these results is a good check on the validity of our equivalent circuit. Dimensional tolerances place the experimental resonant frequency and 3-db bandwidth within the range of experimental error of the determination of the curve of Fig. 2.

Multiple cavity filters can, of course, be constructed either as direct coupled cavities or as "quarter"-wavelength coupled cavities. Fig. 8 presents the response of a three-cavity, three-quarter-wave coupled band-pass filter designed for a maximally-flat amplitude response. The design bandwidth was 500 Mc with the design resonant frequency 11.20 kMc. Experimental values of 525-Mc bandwidth and 11.14 kMc (midway between 3-db points) were obtained. The filter was electroformed from copper upon a series of rectangular blocks on a spindle which were later dissolved out. The curve of Fig. 8 was then obtained with no attempt being made to first tune the filter by means of tuning screws.

Photographs of two twist filters with rotatable center sections are shown in Fig. 9. The filter of Fig. 9(a) is a pure step-twist-junction waveguide filter while that of Fig. 9(b) has 0.400-inch diameter centered circular irises attached to both ends of the rotatable center section. The resonant frequency transmission loss vs 3-db bandwidth curves for these two types are presented in Fig. 10. The resonant frequency vs bandwidth plots (as the center section is rotated) for these two types are presented in Fig. 11. As can be seen in Fig. 11, the resonant frequency of the cavity with circular irises (0.400-inch diameter) is constant out to 755-Mc bandwidth within experimental accuracy of ± 1 Mc at the narrow band-

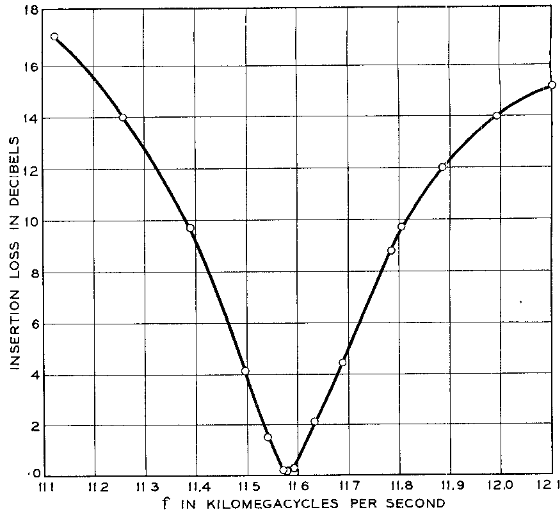


Fig. 7—Transmission characteristic of a single-cavity step-twist-filter.

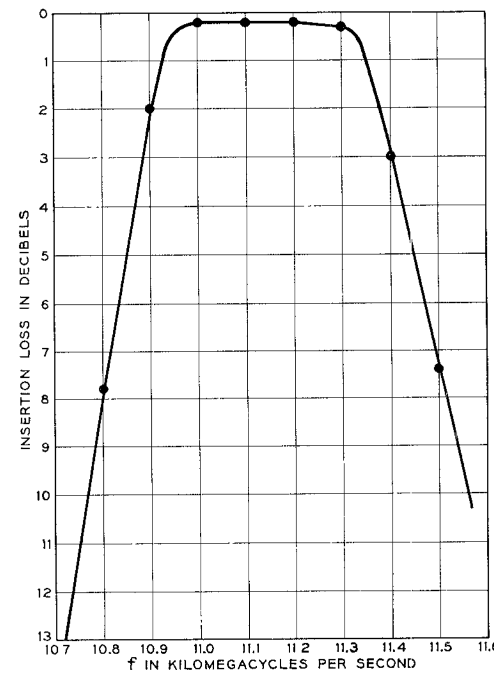
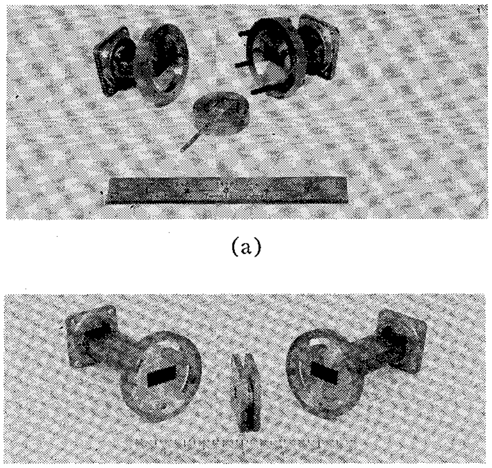


Fig. 8—Transmission characteristic of a three-cavity, three-quarter wavelength coupled, step-twist-junction filter.



(a)

(b)

Fig. 9—(a) Step-twist-junction filter with rotatable center section with no corrective irises. (b) Step-twist-junction filter with rotatable center section with 0.400-inch diameter centered circular irises.

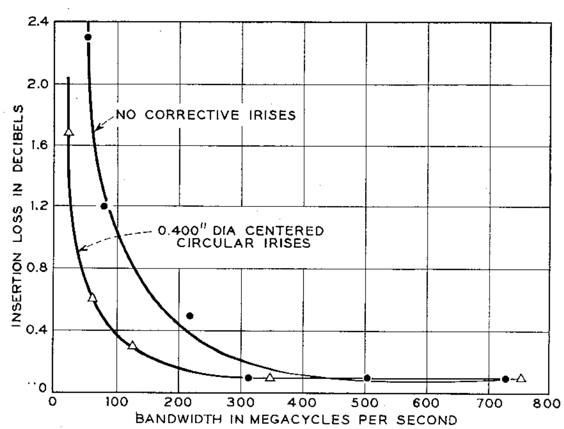


Fig. 10—Insertion loss vs bandwidth for two step-twist-junction filters with rotatable center sections at f_0.

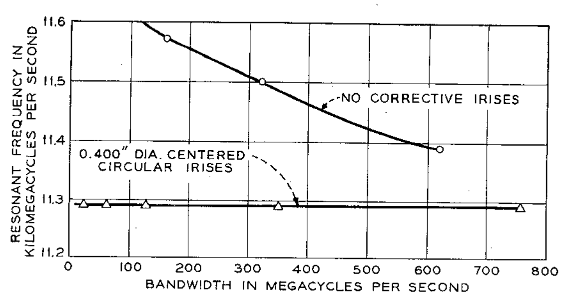


Fig. 11—Resonant frequency vs bandwidth for two step-twist-junction filters with rotatable center sections.

widths, to ±4 Mc at the 755-Mc bandwidth. The resonant frequency is here taken as the point of minimum transmission loss. This 755-Mc bandwidth is the maximum bandwidth that can be obtained with the cavity with circular (0.400-inch diameter) irises attached. The cavity with no irises on the ends of the center section will, of course, produce bandwidths limited only by the useful frequency range of the waveguide, at the expense of a changing resonant frequency with bandwidth and a slightly poorer resonant frequency transmission loss due to a more critical clamping problem for this model. The effects of rectangular and square irises on the resonant frequency vs bandwidth plot were also investigated but these proved inferior to the circular iris result presented in Fig. 11.

Conclusions

Design data has been presented for step-twist-filters in the vicinity of 11.178 kMc. Corrective irises have been described which will keep the resonant frequency of these filters constant (within experimental accuracy) as the bandwidth is varied from a few megacycles to 755 Mc. The addition of tuning screws in the appropriate place will, of course, allow variations of the center frequency of several hundred megacycles (depending upon the required Q of the cavity), and thus these variable bandwidth filters can be quickly and accurately adjusted to a given bandwidth and center frequency with the aid of an X-band sweeper. The design problem of obtaining a given bandwidth with practical tolerances (which in the usual type of filter is not readily adjustable) is thus alleviated.

At the expense of a slightly increased insertion loss, and with the irises attached to the holders rather than the rotatable center section, we may stock a few pieces of X-band guide of different lengths for filter cavities and with appropriate tuning screws we may cover the entire range of the guide in resonant frequency, and all but extremely narrow bandwidths at which the clamped junctions may become troublesome.

The constant frequency filters also allow the interesting possibility of cascading several sections and by adjusting the several θ's and resonant frequencies, actually obtaining any desired response by a direct experimental observation. The angle of rotation can also be servo controlled to provide one with electronically variable filter characteristics.

The inclusion of the first-order frequency sensitivity of step-twist-junctions in the filter design equations (4) and (8) allows one to accurately design broad-band waveguide filters utilizing these junctions.

Appendix

Tests were run on a 70° and an 80° step-twist junction to determine the range of application of (4) and (8). These two values of θ include bandwidths of approximately 1600 to 30 Mc. Values for the admittances of the two twists were obtained at 10.7, 11.2, and 11.7 kMc and were checked against our approximation of Fig. 3(b). For the 70° twist a was constant within experimental error, while B, instead of being a constant, actually varied ±0.3 about its 11.2 kMc value of 2.5 (+0.3 at 10.7 kMc and −0.3 at 11.7 kMc). For the 80° twist a varied ±0.010 about its 11.2 kMc value of 0.046 (−0.010 at 10.7 kMc and +0.010 at 11.7 kMc), while B varied only ± 0.2 about its 11.2 kMc value of 5.8.

For twists of less than 70° a remains constant, while the approximation that B is a constant degenerates; for twists of more than 80° the approximation that a is a constant apparently degenerates, while the approximation that B is a constant improves.

At first glance the apparent 22 per cent error in assuming a equal to a constant for the 80° twist might appear alarming, but it should be remembered that this 22 per cent error is measured over a 1000-Mc range, whereas the 80° twist can be utilized in a filter cavity to produce no more than a 30-Mc bandwidth filter. Actually at the 3-db points, a differs from the 11.2 kMc value by less than 1 per cent. The approximation that a is a constant thus remains valid for twist angles greater than 80°, as well as for twist angles less than 70°.

The approximation that B is a constant, while being very good at twist angles of 80° and higher, is beginning to degenerate for a 70° twist. It is expected that, due to this variation, the design formulas will degenerate for bandwidths much in excess of 10 per cent.

Acknowledgment

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Resonators for Millimeter and Submillimeter Wavelengths*

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Summary—**Further considerations on the mm-wave Fabry-Perot interferometer are presented. Computed Q values for parallel metal plate resonators indicate that at spacings around 2.5 cm, values ranging from 60,000 at 3 mm, to 300,000 at 0.1 mm wavelengths are possible. The plates must, however, be quite flat. These results are important for many investigations, and in particular for mm and sub-mm wave maser research. For the aperture per wavelength ratios possible here, diffraction effects should be small. Consideration is given to using curved reflectors or focused radiation in applications where the fields must be concentrated. For this purpose, re-entrant conical spherical resonators are treated in detail, as regards operation in the TEM mode at high orders of interference. Expressions for the Q and shunt impedance are given, and high values are possible at mm and sub-mm wavelengths. Quasi-optical methods of coupling into and out of such a resonator are proposed, and the higher modes possible in such a resonator are considered. Results indicate that it could have application to the mm-wave generation problem, and that it represents a good resonant cavity for solid state research at mm and sub-mm wavelengths, and for maser applications in particular.**

Introduction

In the region of wavelengths extending downwards from around 1 mm to the long infrared, much important research needs to be done, and many important applications arise. At these wavelengths, conventional cavity resonators become extremely minute, since their dimensions are around one-half wavelength. For some purposes, cavities of larger dimensions, capable of sustaining a number of higher order modes, are possible. This is a difficult procedure, and the difficulties increase with decreasing wavelength for a given size of cavity. Cavities much larger in terms of the wavelength, but which permit mode-free operation, are thus needed. In particular, the development of such a cavity with a suitable interaction gap and new methods of input and output coupling other than conventional waveguides would greatly assist in the development of a primary coherent electronic source for these wavelengths.

Referring to the reflex klystron, which for many purposes is still the most versatile and simplest of microwave tubes, such a cavity must be capable of bunching the electron stream, and hence must possess a suitable interaction gap of small dimensions compared to the wavelength. It also should have a large resonator volume for heat dissipation and a high shunt impedance for efficient electronic interaction. New methods for coupling into and out of the resonator are also necessary. There are other problems, as well, in the design of such tubes for very short wavelengths; another very important one being the provision of an adequate current density at these short wavelengths where the area of the electron beam for efficient interaction with the resonator steadily decreases. This difficulty would certainly be helped by providing larger, more efficient, and more suitable resonators. The required current densities in the resonator gap could possibly be approached with improved cathodes and by the use of suitable magnetic or

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